# Filter Design

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The complementary high-frequency shelving filter can be obtained by replacing the gain *G* in (10) by 1/*G*, which yields the inverse filter, and by multiplying the transfer function (numerator) by *G*, which shifts the magnitude response vertically so that the gain at zero frequency becomes unity (0 dB), and gain *G* is achieved at high frequencies as desired. Additionally, we multiply both the numerator and the denominator by to cancel divisions by *G*.

Comparison with (10) shows that these transfer functions are related as *HHS*(*z*) = *G*/*HLS*(*z*), which means that the pole and the zero of the filter can be interchanged to convert a low shelf to a high shelf or vice versa. Furthermore, a scaling by *G* must be applied. This implies that a cascade of a low and high shelf with the same crossover frequency *wc* and gain *G* will produce a constant gain *G* across all frequencies. Figure 2 shows this for a pair of low- and high-frequency shelving filters.

An alternative way to convert a low shelf filter to a high one is to replace *z* with -*z* in the transfer function (i.e., change the sign of every second coefficient), which turns the response over in frequency so that the zero and Nyquist frequencies are interchanged [A]. Additionally, this low-to-high mapping requires replacing *wc* with *p* - *wc* to restore the crossover frequency.

Figure 2. Magnitude responses of a low (blue) and high (red) shelf filter with a 12-dB gain and 1-kHz crossover frequency, and the cascade of the two filters (black).

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Instead of bandwidth *B*, peak filters are often parameterized using quality factor *Q*. These two terms are related by

*Q* = *wc* /*B*.

Figure 5 presents examples of peak and notch filters with fixed *Q* but varying the other parameters, showing that they are symmetric on the dB scale for *G* and 1/G. It is also seen that when the center frequency is high, such as *fc* = 10kHz, the magnitude response itself becomes asymmetric so that the upper *skirt* (i.e., the magnitude response of a peaking filter on either side of the center frequency) is steeper than the lower one. This feature is caused by the requirement that the digital equalizing filters have a unity gain at the Nyquist limit.

Figure 5. Magnitude responses of peak and notch filters at center frequencies 100 Hz (green), 1000 Hz (blue), and 10000 Hz (red), when *Q* = 1. The gains of the peak and notch filters are complementary, i.e., *G* and 1/*G*, respectively.

[A] Moorer, J.A. The manifold joys of conformal mapping: Applications to digital filtering in the

1145 studio. J. Audio Eng. Soc. 1983, 31, 826–841.

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